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ORIGINAL RESEARCH

A Refined Concept of A Fortiori Arguments for Argumentation Theory

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Abstract
The main goal of the paper is to provide the theoretical model for the a fortiori argument. After a brief history of a fortiori argument (especially in the works of Aristotle, Alexander, Cicero, and Boethius) we propose its general concept, components, and argumentation schemes, its classification, and finally, criteria for assessment. The main reason for this research is that this type of argument receives little attention in contemporary argumentation theory, and consequently critical thinking students have little knowledge of it, and yet, a fortiori arguments (or "arguments from the stronger") are prevalent in both, academic and public, discourse. Therefore, the need to incorporate the concept of a fortiori argument into argumentation theory seems even more crucial. Additionally, we develop the diagrammatic method of assessing the inference in such arguments to finally present four critical questions needed for a critical evaluation of a fortiori argument.

Keywords a fortiori argument · A simili · Analogy · Argument from the stronger · Argumentation scheme · Comparative argument · From the less · From the more

1 Introduction

A fortiori arguments (hereinafter: AFAs) are commonly encountered in the humanities, social sciences, and public discourse. The expression "a fortiori" functions descriptively and adverbially, indicating an argument whose conclusion is drawn in a particular way—a fortiori. But what does it mean to say that a conclusion follows a fortiori? In the most general sense, it could be said that a fortiori reasoning is

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based on the comparison of objects: specifically, the fact that two objects are positioned as more and less on some comparative scale is pivotal for inferring a conclusion.[1] Two intuitive examples can help illustrate this:

(a) A speed of 6 km/s is not enough to launch an object into the atmosphere. Therefore, all the more, a speed of 4 km/s is not enough to launch an object into the atmosphere.
(b) A man can lift a weight of 40 kg, therefore, all the more, the man could lift a weight of 20 kg.

Both arguments seem plausible due to the comparisons between two pairs of objects: faster/slower velocities and heavier/lighter weights. In argument (A), a conclusion about a slower velocity not being fast enough to carry an object into the atmosphere is deduced based on the fact that a faster velocity was not fast enough. In other words, if a faster velocity is not sufficient, a slower one is even less so. In argument (B), a conclusion about a man's ability to lift a lighter weight is deduced based on the fact that he was able to lift a heavier weight. Thus, if he could lift a heavier weight, he could certainly lift a lighter one.

Despite the attested usage of AFAs since classical antiquity, the concept receives little attention in the contemporary manuals of critical thinking and argumentation theory. For instance, there is no mention of them in Walton, Reed, and Macagno's monumental work, *Argumentation Schemes* (2008), nor in some of the most popular handbooks on logic and critical thinking. To name just a few: Hurley (2000), Johnson and Anthony Blair (2006), Fischer (2011), Groarke and Tindale (2013), Woods (2013), van Eemeren et al. (2014), and Govier (2018).[2]

Nonetheless, this does not mean all scholars disregard the concept AFA completely. There is, in fact, disagreement about the fundamental nature of AFAs: should they be viewed as truth-preserving or merely probable? If they are inherently truth-preserving, is there a quick method available for proving their validity? Conversely, if they do not necessitate a conclusion, how should we evaluate their logical and persuasive strength?[3]

The primary aim of this paper is to present a theoretical framework for AFAs that can aid in their identification, analysis, and assessment. In the next section, an overview of comparative reasoning in historical sources is provided. Then, we introduce Avi Sion's significant achievements in a fortiori logic and adapt them into the broader context of argumentation theory, where AFAs could be treated as a distinct argumentation scheme. After defining AFA, we develop Sion's idea of checking the

[1] See: Fabião (2023, 178), d'Almeida (2017, 204), Valencia Martinez (2016, 4), and Marraud (2014, 106–7).

[2] A few exceptions in Anglophone literature are: Rigotti and Greco (2019, 84. 112), Rubinelli (2009, 18–21; 133–40). Considering non-Anglophone academic world, it is worth to mention the following findings: Dutch: (Garssen 1997, 103–05; 114), French: (Perelman and Olberechts-Tyteca 1971, 337–45; Doury 2016, 98–100); Polish: (Szymanek 2004, 62–64), German: Kienpointner (1992a, 178–88; 1992b, 299–303).

[3] Fabião (2023), Miron (2021), and Valencia Martinez (2016) hold that AFAs are not truth-preserving, whereas d'Almeida (2017), Sion (2013), Alexy (2009) argue that AFAs necessitate their conclusions.

formal validity of a fortiori reasoning into an informal method of critical questions. We also show how the concept of AFA may be incorporated into the broader concept of arguments from analogy.

2 Historical Approaches to A Fortiori Arguments

Throughout antiquity, arguments based on comparison were both used in argumentation and studied from a theoretical point of view. Clear examples of reasoning based on comparison are evident across Plato's and Aristotle's works, with Aristotle being the first to undertake a study of such arguments.[4] In ancient Greece, they became known as arguments "from the more and less" (ἀπὸ τοῦ μᾶλλον καὶ ἧττον), and in the later Latin tradition, as arguments "from the comparison of the greater and lesser" (ex comparatione maiorum et minorum).[5]

Aristotle's treatment of arguments "from the more and less" in *Topics* and *Rhetoric* significantly influenced further approaches to these types of arguments. Not being discussed as part of demonstrative argumentation, arguments "from the more and less" were not subjected to the rigorous scrutiny of Aristotle's formal logical theory and demonstrative science developed in the *Analytics*. Instead, they were introduced, classified, and illustrated as part of dialectical and rhetorical argumentation, where different logical, epistemic, and pragmatic standards apply.[6]

After Aristotle, Alexander of Aphrodisias attempted to relate these arguments to syllogisms from hypotheses but did not expand or refine Aristotle's classification. By the time of Cicero, reasoning based on comparison had been somewhat simplified and modified for public speaking and particularly for courtroom purposes. In the twilight of antiquity, as evident from the works of Boethius, argumentum ex comparatione remained listed as just one of the many types of arguments suitable for concluding various subject matters in public speech settings. The following historical inquiry is aimed to demonstrate that in the ancient tradition AFAs were understood as topical arguments within the broader framework of argumentation theory. Thus, the incorporation of AFAs into argumentation theory is not solely a reminder what the AFAs are, but should be seen as a continuation of ancient tradition.

[4] For a catalog of a fortiori reasoning in historical sources see Sion (2013, 561–70).

[5] Arguments ex comparatione besides arguments "from the more" and "from the less" often include arguments "from the equal" traditionally known as a pari arguments (see Ari. *Rhet*. II.23, 1397b19-29, *Top*. II.10, 115a25 Cic. *Top*. 22, 24–29, and Boeth. *De top. diff*. 1199C34–40). These can be regarded as comparative arguments since asserting that two items are equal presupposes that they have been compared on some comparative scale. In the historic overview, we shall omit these variants of comparative arguments since their logical force is not based on comparative difference ("much more" or "much less") but on comparative equality ("equally so"). For a different view see Sion (1995, 45) who treats a pari argument as a special type of a fortiori argument.

[6] See Cicero, *Top* 6–8, Boethius, in *Cic. Top. comm*. 273–77, and *De top. diff*. 1173C1-8 for a distinction between *ars inveniendi* and *ars iudicandi*. It is also worth to emphasise that we will treat AFAs—unlike Sion—as sufficiently general and plausible, but potentially defeasible, i.e. not exclusively as truth preserving in the strict sense.

2.1 Classification of Reasoning Based on Comparison in Antiquity

In Aristotle's *Rhetoric* (II.23, 1397b12-20), two forms of arguments "from the more and less" can be recognised. However, in his *Topics* (II.10, 114b37-115a14), Aristotle identifies four major templates, each of which enables one constructive and one destructive argument. Apart from Alexander of Aphrodisias, who, in his commentary on Aristotle's Topics, extensively illustrates all four templates (see Alex. in *Ari. Top. comm.* 204, 28–208, 11), the later tradition seems to follow Aristotle's *Rhetoric* rather than *Topics* and sticks only to the basic difference between arguments traditionally recognised as a maiore ad minus (from greater to lesser) and a minore ad maius (from lesser to greater).[7]

Anyhow, the four templates for making arguments "from the more and less" identified in Topics are based on the number of terms: arguments with (i) one subject and one predicate; (ii) two subjects and one predicate; (iii) one subject and two predicates; (iv) two subjects and two predicates. Alexander follows Aristotle (in *Ari. Top. comm.* 204,28-205,5) and, in each template, recognises one argument "from the more" and one argument "from the less" with one always being constructive and the other destructive, depending on the dialectical move one wants to make regarding the dialectical question.

Type (I): Arguments with one subject and one predicate

I. If one attribute is predicated of one subject, then:

I.1. If an increase in the accident follows an increase in the subject, the accident belongs;
I.2. If it does not follow, the accident does not belong (see *Topics* II.10., 114b37-115a5).

Constructive argument "from the more":

1. More pleasure means more good than less pleasure.
2. Therefore, pleasure is good (by I.1).

Destructive argument "from the less":

1. More pleasure does not mean more good than less pleasure.
2. Therefore, pleasure is not good (by I.2).

Type (II): Arguments with two subjects and one predicate

II. If one predicate is attributed to two subjects, then:

[7] See Reinhardt (2003, 247–48), Valencia Martinez (2016, 7–8), and Sion (2013, 99–101; 109-11). See also Miron (2021, 9), Fabião (2023, 180), and d'Almeida (2017, 211) for a contemporary reception of a maiore and a minore in legal discourse.

II.1. If a predicate does not belong to a subject to which it is more likely to belong, then the predicate does not belong to a subject to which it is less likely to belong;
II.2. If a predicate does belong to a subject to which it is less likely to belong, then it also belongs to a subject to which it is more likely to belong (see *Topics* II.10, 115a6-11).

Destructive argument "from the more":

1. The good does not belong to health.
2. The good is more likely to belong to health than to richness.
3. Therefore, good does not belong to richness [by II.1].

Constructive argument "from the less":

1. The good belongs to doing gymnastics.
2. Doing gymnastics is less good than richness.
3. Therefore, the good belongs to richness [by II.2].

It is worth noting that Alexander (in *Ari. Top. comm*. 205, 22-25 and 207, 10) is aware of two remaining scenarios where the premise does not lead to the conclusion about the remaining of the two compared subjects: "constructive from the more," and "destructive from the less."[8] First, the fact that a property *P* does belong to subject *S1*, to which it is more likely to belong than to subject *S2*, does not lead to a conclusion about the property belonging to *S2*. Second, the fact that a property *P* does not belong to subject *S1*, to which it is less likely to belong than to subject *S2*, does not lead to the conclusion about *P*'s not belonging to *S2*. More simply, arguments "from the more" are only destructive, whereas arguments "from the less" are only constructive.

Type (III): Arguments with one subject and two predicates

III. If two predicates are attributed to one subject, then:

III.1 If a predicate that is more likely to belong to a subject does not belong, neither does the predicate that is less likely to belong;
III.2 If a predicate that is less likely to belong does belong to a subject, so also does the predicate that is more likely to belong (see *Topics* II.10, 115a11–14).

Destructive argument "from the more" (this is also Alexander's example):

[8] See also Aristotle (*Top* III.6, 119b16-119b30) and Sion (2013, 103).

1. Being useful does not belong to learning.
2. Being useful is more likely to belong to learning than being pleasant [belongs to learning].
3. Therefore, being pleasant does not belong to learning (by III.1).

Constructive argument "from the less":

1. Being fair is less just than being philanthropic.
2. Being fair is just.
3. Therefore, being philanthropic is just too (by III.2).

As in the previous case, arguments "from the more" are only destructive, whereas arguments "from the less" are only constructive.

Type (IV): Arguments with two subjects and two predicates

IV. If two predicates are attributed to two subjects, then:

IV.1 If the one which is more usually thought to belong to the one subject does not belong, neither does the remaining predicate belong to the remaining subject;
IV.2 If the one which is less usually thought to belong to the one subject does belong, so too does the remaining predicate to the remaining subject (see *Topics* II.10, 115a16–21).

Destructive argument "from the more":

1. The profligate person is generous to a higher degree than the stingy person is moderate.
2. The profligate person is not generous.
3. Therefore, the stingy person is not moderate either (by IV.1).

Constructive argument "from the less":

1. The pleasure is good to a lesser degree than toil is bad.
2. Pleasure is good.
3. Therefore, the toil is bad (by IV.2).

In the case of arguments of type (IV), as well as before, arguments "from the more" are destructive and arguments "from the less" are constructive. Note, however, that the inference of the conclusion in type (IV) does not have to be based solely on comparison: both conclusions follow from their respective premises employing the argument "from contraries" warranted by the principle: if S and P admit contraries, then if the contrary of S is the contrary of P, S is P; and if the contrary of S is not the contrary of P, then S is not P. As it will be shown later

on, not all of these types are a fortiori arguments *par excellence*—for this it needs to include four terms, from which there are two subjects and one predicate, even though usually not all of them are expressed explicitly.

In contrast to Aristotle and Alexander, Cicero and Boethius recognise only two forms of arguments "from the more and less," and what is intriguing, this simplified approach was favoured for a long time in logic and rhetoric. According to Cicero (*Topica*, 22,21–24), the two rules of licensing inference in arguments *ex comparatione* are:

From the greater: what holds good in the greater, holds good in the lesser:

1. Boundaries are not regulated in the city.
2. Therefore, neither should water be excluded in the city.

From the lesser: what holds good in the lesser holds good in the greater:

1. The water is excluded in the city.
2. Therefore, the boundaries should be regulated in the city as well.

The concise formulation of these principles is noteworthy. It is not immediately clear which is constructive and which is destructive, as both establish that something holds good in one case based on the fact that it holds good in the other. Furthermore, it is not transparent how premises and conclusions fit the principles that warrant these inferences, since "major" and "minor" here do not merely denote the likelihood of predication as in Aristotle and Alexander, but rather indicate the relationship between two legal actions on a comparative scale relevant to whether these actions should occur or not (Reinhardt 2003, 247–48; Sion 2013, 110–11). Put simply, an argument "from the more" (or "from the less") in the case of Cicero does not necessarily start with a premise about the more (or less) likely predication, but rather with a premise stating *res maior* (or *minor*).

Compare "if 6 km/s is not fast enough for reaching cosmic speed, much less is 4 km/s" and "if 6 km/s can be reached, much more could be 4 km/s": the former starts with a premise concerned with the more likely predication and includes *res maior* (6 km/s is greater than 4 km/s). The latter begins with the less likely predication, but—if the same comparative scale is maintained—still involves *res maior*.

This shift in criteria for categorising an argument as "from the more" or "from the less" likely does not result from Cicero's theoretical insights into the relevance of comparative scales in reasoning *ex comparatione* (see further below and in the next section 3). Instead, Cicero appears to operate within a discourse concerned with reasons for undertaking specific legal actions. Regrettably, without detailed knowledge of Roman legislation, it is hard to understand on what ground two legal cases could be compared, and thus, on what ground a certain

legal action or measure in one case (*major* or *minor*) should entail a certain legal action or measure in the other one (*major* or *minor*).[9]

While Cicero's interpretation leaves some ambiguities, Boethius' treatment of arguments from comparison focuses on the objects being compared and the scale by which they can be seen as more or less.

Boethius distinguishes between a comparative argument "from the more," where the greater is compared to the lesser, and a comparative argument "from the less," where the lesser is compared to the greater (*De top. diff.* 1199B-C). Boethius's *a maiore* and *a minore* are based on the following principles:

From the greater: what holds good in the greater holds good in the lesser (1199 B24-C33):

1. The person *A* committed more severe crimes than person *B*.
2. The person *A* was pardoned.
3. Therefore, the person *B* should be pardoned.

From the lesser: what holds good in the lesser holds good in the greater (1199 C34–40):

1. Gaius Gracchus committed less severe crimes than Catiline.
2. Gaius Gracchus was punished.
3. Therefore, Catiline should be punished.

Note that in both arguments the same comparative scale is assumed: objects are compared as more and less concerning the severity of crimes they committed. However, the interpretation of "more" and "less" differs based on the context of the likelihood of receiving a pardon or punishment. The one who committed more severe crimes is considered less deserving to be pardoned, but more deserving to be punished. Conversely, the one who committed less severe crimes is more deserving to be pardoned but less deserving to be punished. This contrast may suggest that Boethius did not outline two distinct forms of arguments, but rather the same form applied to polar predicates (granting a pardon and punishment). Specifically, he presented two constructive arguments "from the less 'likely predication"' without recognising that the objects in these two cases are scaled in opposite ways.[10]

2.2 The Nature of Reasoning from Comparison in Antiquity

Comparative arguments in antiquity belong to the category of topical arguments, which are based on the so-called topical principles. The notion of *topos* plays a vital role in ancient logical theory. Starting with Aristotle, *topoi* are strategies or

[9] See Reinhardt (2003, 250), Sion (2013, 109–10) and Boethius (*Cic. Top. comm* 308) for a possible reconstruction of this particular case.

[10] Boethius draws the difference between destructive from the more and constructive from the less based on the likelihood of predication (*De top. diff.* 1191A).

instructions grounded on principles that one can rely on to produce cogent and persuasive arguments in dialectical debate and public speech. From late antiquity onwards, *topoi* were conceived as so-called maximal propositions and their *differentiae* an arguer needs to recognise to pick an appropriate maximal proposition as the basis for their argument.[11]

Despite the evolving notion of *topos* from Aristotle to Boethius, the dual function of *topos* as both the source of an argument and its warrant has been preserved. The *topos* related to comparative arguments are, just like any other *topos*, best represented by a semi-formal "if-then" sentence (Toulmin's "warrant") that serves as a pattern for presenting arguments. An arguer's main task is to show that the secured premises and conclusion fit the structure of the "if-then" topical principle. If this is shown, the guaranteeing power of a topical principle ensures that the interlocutor and audience would accept an arguer's standpoint based on the premises they allowed. But on what grounds does the *topos* "from the more and the less" warrant the inference?

It appears that scholars in antiquity were aware that some "if-then" sentences are true in virtue of their form, while others are true precisely because of the content in their antecedent and consequent (see Cicero, *Topica* 53, 1–25 and Boethius, *De syllogismis hypotheticis* 835B). Even though the fact that *topoi* as stated are material relations, the application of some *topoi* can result in necessary connections. These are usually found in arguments derived from *topoi* based on definitional aspects of general logical, semantical, and metasemantical notions. The topical principle underlying comparative argument could be conceived as expressing frequent but non-necessary relations (Macagno 2015, 189). Regardless, the standards of argumentation context are such that an interlocutor or audience would be sufficiently convinced by the conclusion of a topical argument, whether it is grounded in a *topos* expressing necessary connections or the one expressing only frequent connections as long as they accept the premise and see that it fits topical principle.

> What the group sees as a fairly high degree of likelihood, or normative or logical validity is sufficient. To take an example: It is by no means impossible that a weaker man, by means of special training, can lift more than a strong man, but for the audience, this exception does not negate the general acceptability of the principle that it is the more likely case that the stronger man can lift a heavier load (Braet 2005, 75).

The potentially defeasible nature of topical principles in general leads some contemporary scholars to think that *topoi* are forerunners of argumentation schemes.[12] Consequently, traditional comparative arguments could be reconstructed in the

[11] For a short overview of the notion of *topos* from Aristotle to Boethius see (Stump 1978, 205–14). Detailed (and often competing) accounts of Aristotle's approach to topical reasoning can be found in: (Smith 1997; Primavesi 1996; Slomkowski 1997; Braet 2005). See Castelli (2020) and Stump (1978; 1988) for Alexander, Cicero, and Boethius. A short overview of topical arguments in antiquity through the lens of contemporary argumentation theory is given in: (Macagno et al. 2017; Rubinelli 2009; Walton et al. 2008).

[12] See particularly (Macagno and Walton 2011; Braet 2005, 75; Rubinelli 2009, 14; Macagno et al. 2017; Walton et al. 2008; Rigotti and Greco 2019).

defeasible *modus ponens* patterns starting with the topical principles as a defeasible major premise (see Valencia Martinez (2016)).[13] However, we should keep in mind that in the ancient tradition "if-then" topical principles, despite some of them not resulting in necessary connection, are sufficiently general and plausible to serve as starting points for dialectical and rhetorical debate (see Boeth. *De top. diff.* 1185C8–11). In other words, they can attain the status of *endoxa* or commonly accepted opinions that an audience would not question, making the arguer's primary task to secure such premises that would fit the *topos*.

3 Classification and Validity of A Fortiori Arguments

An extensive study of AFA was done by Avi Sion, initially in *Judaic Logic* (1995), and later in the book *A Fortiori Logic: Innovations, History and Assessments* (2013). He defined AFA as "4-terms-syllogism" including major (P), minor (Q), middle (R), and subsidiary (S) terms, but the premises with the major and middle term are quite often unstated. Sion proposed a sophisticated analysis of AFAs—he distinguished 2 forms, 4 structures, 2 polarities, and 2 orientations of such arguments. We will use these categories to build the classification of AFAs. The first division could be made due to the *form*—consider the two following examples:

(c) If my Oyster card is transferable to other adults, and usually adults pay more than students for public transport, therefore my Oyster card can be used even more by the students.
(d) In 2016 52% of Brits wanted to leave the European Union, and it was enough for democratic decision-making, so the UK left the EU. Nowadays 64% of Brits think that immigration has been too high in the last 10 years, so the UK should even more restrict its immigration policy (YouGov 2024).

These examples of AFA manifest the first difference between AFAs regarding the form: the former argument is "copulative," and the latter is "implicational" in form. The copulative form concerns terms, whereas the implicational form deals with propositions. Let us consider both examples more carefully. So example (c) is the following:

1. Students (P) are allowed to pay less for public transport (R) than adults (Q).
2. Adults (Q), who must pay regular tickets for public transport (R), can use any regular Oyster card (S).
3. All the more, students (P) are allowed to use a regular Oyster card (S).

The terms in example (c) are the following:

[13] For a different approach to AFAs in general, but still assuming the broad notion of an argumentation scheme, see Marraud (2014). Sion (1995; 2013) on the other hand, assumes that comparative reasoning in antiquity is designed to be truth-preserving, and the same is maintained regarding AFAs (see the next section).

P – students
R – obligation to pay for public transport
Q – adults
S – allowance to use a regular Oyster card

So the scheme reads[14]

1. *P* is more *R* than *Q* is *R*.
2. *Q* is *R* enough to be *S*.
3. *All the more*, *P* is *R* enough to be *S* (or simply: *P* is *S*).[15]

As stressed in the scheme above, this form involves predication—to describe the relations between the terms it uses "to be" predicate. In the (d) example it will be quite different:

1. 64% of society's approval in democratic decision-making is more than 52%;
2. In 2016, 52% of British society's approval was enough to make a change (and the UK left the EU);
3. *All the more*, 64% of British society's approval to change the immigration policy is enough to make a change (and restrict the policy).

The terms in example (d) are the following:

P – The United Kingdom left the European Union.
Q – 52% of Brits wanted the United Kingdom to leave the European Union.
R – 64% of Brits think that immigration has been too high in the last 10 years.
S – The United Kingdom should restrict its immigration policy.

Thus, the scheme reads:

1. *P* implies more *R* than *Q* implies *R*.
2. *Q* implies enough *R* to imply *S*.
3. All the more, *P* implies enough *R* to imply *S*.

This brings us to the first distinction, which Sion describes as a copulative vs. implicational form (1993, 43), and allows us to build the first-level division, disjunctive and complete, of AFA (Fig. 1):

[14] One could try to formalize this as [R(p)> R(q)] ∧ [R(q) *is enough to*: S(q)] → [R(p) *is enough to* S(p)], but this would require second-order logic which might be rather an obstacle on the field of argument theory, therefore we will skip it.

[15] This scheme is built in a full and ideal form—most AFAs in common usage skip some parts of it. We will discuss this problem later on, but so far, we will provide the full scheme following—with some modifications - Sion.

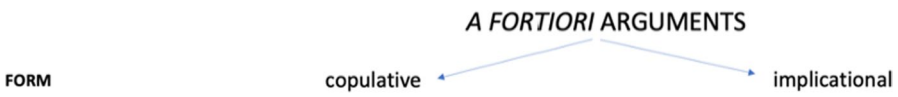


Fig. 1 First-level division of AFAs

In each of these two forms, we can differentiate two different structures—which continues the classification of AFA—depending on whether the extreme terms, *P* and *Q* (the terms that vary on the intensity of having the property *R*), are subjects or predicates. In subjective structure (as in (c) example) they have the role of subjects in all three propositions, and in predicative structure, they would have a role of predicates, e.g.:

(e) You need more determination to do a triathlon than rowing, and definitely, the Ironman race requires more determination and strength than a triathlon. So, the Ironman requires even more determination than rowing.

The standardised and listed argument reads:

1. More determination is required to do a triathlon than rowing.
2. The Ironman race requires more determination than triathlon.
3. *All the more*, the Ironman race requires more determination than rowing.

So the scheme for predicative structure is following—it can be easily observed that it differs from the aforementioned scheme in example (c):

1. More *R* is required to be *P* than to be *Q*.
2. *S* is *R* enough to be *P*.
3. All the more, *S* is *R* enough to be *Q*.

Accordingly, for the implicational form, we can build two structures, antecedental and consequental. The former was already formulated in the example (d), so let us consider the structure only the for latter:

1. More *R* is required to imply *P* than to imply *Q*.
2. *S* implies enough *R* to imply *P*.
3. All the more, *S* implies enough *R* to imply *Q*.

Since these two structures are the only ones which are possible in each form, and they are irreducible to each other, the division continues to be complete and disjunctive (Fig. 2):

Now, the third category that needs to be included is orientation. As it was shown in the historical overview, AFAs can either be of two kinds: *a minori ad maius* and *a maiori ad minus* depending on whether the reasoning moves from the minor (or

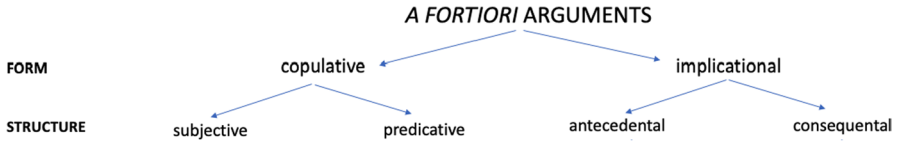


Fig. 2 Second-level division of AFAs

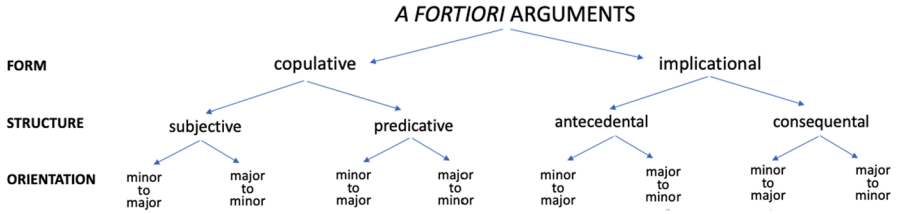


Fig. 3 Third-level division of AFAs

major) term in premise to major (or minor) term in conclusion. Since these are two options, which are also disjunctive we develop the third-level division (Fig. 3):

Finally, the fourth category that should be brought out is the polarity—this is quite often in argumentation theory that we distinguish positive and negative variants of the argument.[16] For each of the arguments in this classification, we could formulate a positive variant. Let us take the subjective structure with major to minor orientation—the negative variant would read:

1. *P* is more *R* than *Q* is *R*.
2. *P* is *R* not enough to be *S*.
3. All the more, *Q* is *R* not enough to be *S*.

Accordingly, we can build it to every argument, so we receive 16 possibilities (Fig. 4):

However, not every variant will be valid—consider a similar argument (a subjective structure with major to minor orientation), but in a positive variant:

1. *P* is more *R* than *Q* is *R*.
2. *P* is *R* enough to be *S*.
3. All the more, *Q* is *R* enough to be *S*.

After looking further into this scheme, one can see that it is not valid (all the previous schemes were, indeed, valid). We can check validity in a very simple, graphic way (just as we do with Venn's diagrams for syllogism). Let us first note that all

[16] See for example positive and negative variants of *ad consequentiam* (Walton 2008, 26; 2013, 102) or the positive and negative variants of *ex concessis* arguments (Pruś 2023; Ervas and Mosca 2024).

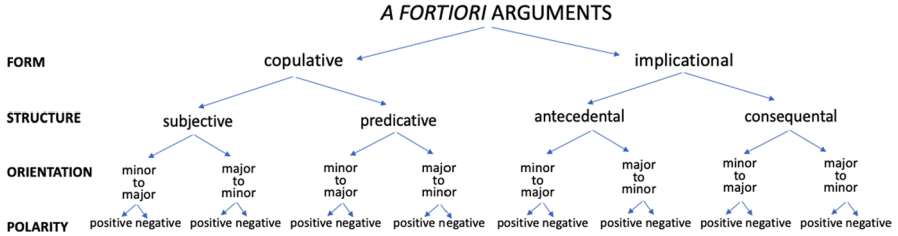
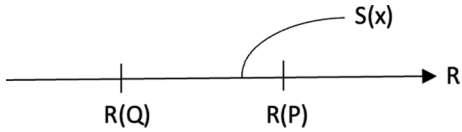


Fig. 4 Fourth-level division of AFAs

Fig. 5 Diagram invalidating a subjective major-to-minor positive reasoning



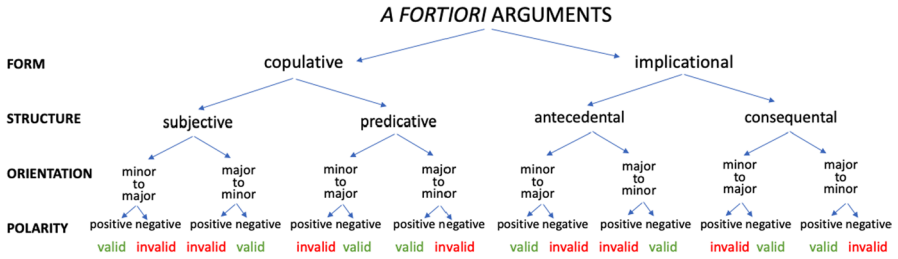


Fig. 6 Classification of AFAs with valid schemes

three terms, *P*, *Q*, *S* and just the arguments of the middle term, *R*, which is a predicate to them, and it varies these terms quantitatively—in other words *R* is a property which objects (or propositions), *P*, *Q*, *S*, have to a certain degree. Inspired by Sion,[17] who also used the x-axis to express the relation between the terms [*P, Q, R, S*] we may build the following diagram, on which we mark two premises—if the diagram precisely indicates the conclusion, the reasoning is valid, if not—invalid. Thus we see that it is possible to include the premises on the diagram in a way that makes the conclusion false—which means that the reasoning is not valid (Fig. 5):

If we analysed all the 16 schemes with this method, we would find only 8 valid schemes—each structure (subjective, predicative, antecedental, and consequental) has 2 valid schemes, each with a different orientation (minor-to-major and major-to-minor), and each with a different polarity (positive and negative)[18] (Fig. 6):

[17] Sion dedicates a part of his book to validating various moods of AFA (1995, 46–52; 2013, 17–25), however, he uses the x-axis differently—simply by putting the letter on the axis. It proves much better, however, to mark the whole range to show the relation between the ranges more precisely.

[18] There are many interesting patterns already visible in the diagram, which can be analysed more carefully, e.g. predicative and consequental schemes, due to the inverted sentence structure, have the oppo-

Table 1 Classification of AFAs with all schemes

FORM	STRUCTURE	ORIENTATION	POLARITY	
			POSITIVE	NEGATIVE
COPULATIVE	SUBJECTIVE	minor to major	P is more R than Q is R. Q is R enough to be S. All the more, P is R enough to be S.	P is more R than Q is R. Q is R *not* enough to be S. All the more, P is R *not* enough to be S.
		major to minor	P is more R than Q is R. P is R enough to be S. All the more, Q is R enough to be S.	P is more R than Q is R. P is R *not* enough to be S. All the more, Q is R *not* enough to be S.
	PREDICATIVE	minor to major	More R is required to be P than to be Q. S is R enough to be Q. All the more, S is R enough to be P.	More R is required to be P than to be Q. S is R *not* enough to be Q. All the more, S is R *not* enough to be P.
		major to minor	More R is required to be P than to be Q. S is R enough to be P. All the more, S is R enough to be Q.	More R is required to be P than to be Q. S is R *not* enough to be P. All the more, S is R *not* enough to be Q.
IMPLICATIONAL	ANTECEDENTAL	minor to major	P implies more R than Q implies R. Q implies enough R to imply S. All the more, P implies enough R to imply S.	P implies more R than Q implies R. Q does *not* imply enough R to imply S. All the more, P does *not* imply enough R to imply S.
		major to minor	P implies more R than Q implies R. P implies enough R to imply S. All the more, Q implies enough R to imply S.	P implies more R than Q implies R. P does *not* imply enough R to imply S. All the more, Q does *not* imply enough R to imply S.
	CONSEQUENTAL	minor to major	More R is required to imply P than to imply Q. S implies enough R to imply Q. All the more, S implies enough R to imply P.	More R is required to imply P than to imply Q. S does *not* imply enough R to imply Q. All the more, S does *not* imply enough R to imply P.
		major to minor	More R is required to imply P than to imply Q. S implies enough R to imply P. All the more, S implies enough R to imply Q.	More R is required to imply P than to imply Q. S does *not* imply enough R to imply P. All the more, S does *not* imply enough R to imply Q.

Let us now gather all of these schemes with their schemes in the table (Table 1):

To make the references easier, we will follow Sion's shortcuts: *s*—subjective, *p*—predicative, *a*—antecentendal, and *c*–consequential. Furthermore, we will use "-" for negative polarity and "+" for positive polarity. Therefore, there are 8 valid schemes of AFA: *+s, -s, +p, -p, +a, -a, +c, -c* (highlighted in green), and 8 invalid schemes (the schemes in red), all of them presented at the Fig. 7:

To complete our classification and analysis of AFA's possible forms, structures, orientations and polarities, we need to consider one last thing—that is, so to say, "a grammar style." Let us consider the following:

- *P* is more *R* than *Q* is *R*.
- *Q* is less *R* than *P* is *R*.

Footnote 18 (continued)
site valid schemes as subjective and antecedental. For further analysis of these patterns, see Sion (2013, 16–34).

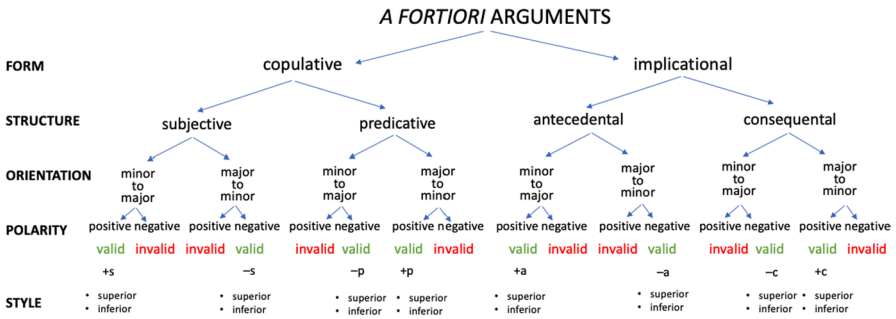


Fig. 7 Classification of AFAs with valid schemes and styles

Although they may seem different, they are not—the only thing that changed is the order of the terms, just as the sign greater-than has changed into less-than. Using a different word order with the different comparison operators does not constitute a separate scheme or mood—it just makes analysis a bit foxier. So far, we have used only the former, called "superior" (where the latter is called "inferior").[19]

Having classified the valid schemes of AFA, along with the method of determining its formal validity, we may now move to incorporate the results into argumentation theory.

4 A Fortiori Arguments for Argumentation Theory

The concept of AFA just presented—although it is well-elaborated—can hardly be used by the average "user of argumentation theory," who follows Walton's method of argumentation schemes (for identification of the argument type) and critical questions (for evaluation of the argument).

The idea is rather simple—in the book *Argumentation Schemes* Walton, Macagno and Reed provide us with 60 schemes: a few basic (e.g. argument from authority), and then more specific (e.g. from expert opinion, from witness testimony, from position to know etc.) types of argument. These are characterised by the *argumentation schemes*:

> Argumentation schemes are forms of argument (structures of inference) that represent structures of common types of arguments used in everyday discourse, as well as in special contexts like those of legal argumentation and scientific argumentation. They include the deductive and inductive forms of argument that we are already so familiar with in logic. However, they also represent forms of argument that are neither deductive nor inductive, but that fall into a third category, sometimes called defeasible, presumptive, or abductive.

[19] Sion distinguishes also the third style, "egalitarian" (1995, 44): "P is as much R as Q"—this, however, fits perfectly to the scheme of the argument from an analogy—as it will be demonstrated—so it should not be considered as AFA.

> Such an argument may not be very strong by itself, but may be strong enough to provide evidence to warrant rational acceptance of its conclusion, given that its premises are acceptable (2008, 1).

Knowing the scheme of the argument of a certain type allows users of argumentation theory (who could know little of logic) to identify the specific type in the given argument and accordingly formulate critical questions to it. These are the unique criteria formulated separately for each type of argument.[20] Our goal is to incorporate the concept of AFA into argumentation theory, that is, to formulate a generic scheme (or schemes)—that helps to identify this type of argument—and critical questions—which allow us to assess the argument without detailed investigation of its structure and inferences (as we did on the diagrams).

4.1 Generic Scheme of AFA

In the previous analysis, we have formulated, following Sion, 8 schemes with valid reasoning *+s, -s, +p, -p, +a, -a, +c, -c*. However, for the application to argumentation theory, they can be somehow merged: differences between the subjective & antecedental and predicative & consequental are purely grammatical, and all *p*-schemes and *c*-schemes can be easily transposed into accordingly *s*-schemes and *a*-schemes. Additionally, since predication in *s*-schemes can also be viewed as a special case of implication,[21] all schemes are reducible to into *a*-scheme. The schemes which cannot be reduced to each other are these with different orientations: major-to-minor and minor-to-major. Therefore, if we are about to express all the arguments in the most generic scheme, we have to use two antecedental schemes (*+a* and *-a*)—following the naming convention we shall refer to them as "constructive" and "destructive" form[22]

[20] To give a brief example: John hears the argument: "Since Einstein said that everything is relative, there are no objective values", and recognises the scheme of argument from authority: "Expert E asserts that X, therefore X is true," and thus John asks himself 4 critical questions: "(i) Did E asserted X?; (ii) Is the authority of E in the field relevant to the issue discussed? (iii) Is E biased?; (iv) Do other experts from the field also think that?". In this way, John may very easily make use of the theory of arguments from authority barely knowing about it.

[21] As Sion explains: "The logical relationship between 'is' and 'implies' is well known. X 'is' Y, in class-logic terminology, if it is subsumed/included by Y, which does not preclude other things also being Y. X 'implies' Y, if it cannot exist/occur without Y also existing/occurring, even if as may happen it is not Y. Thus, if X 'is' Y, it also 'implies' Y; but if X 'implies' Y, it does not follow that it 'is' Y. In other words, 'is' implies (but is not implied by) 'implies'; 'implies' is a broader more generic concept, which covers but is not limited to 'is,' a narrower more specific concept" (1995, 44).

[22] We could simply refer to them as "minor to major/major to minor" a fortiori: argument or respectively in Latin: "a maiore ad minus / a minore ad maius," yet "constructive/destructive" seems better than each of these propositions: it is simpler and far more intuitively describes the nature of the argument (to construct—presenting some positive argument; to destruct—disagreeing with a certain idea). Alternatively, we could refer to them according to the polarity as "positive/negative" versions of the AFA, meaning their logical form (affirmation or negation) not their role in the given discussion (positive argument might be used to refute just as negative argument might be used to support the conclusion). It is also worth to mention that many argument types has two (or more) "subschemes," e.g.: arguments from authority (Koszowy and Walton 2019, 154), ad hominem arguments (Pruś 2023, 55–56), arguments from consequences (Szymanek and Wieczorek 2020, 63).

A fortiori argument—constructive scheme (positive):

P1. *P* implies more *R* than *Q* implies *R*;
P2. *Q* implies enough *R* to imply *s*;
C. *All the more*, *P* implies enough *R* to imply *s*.

A fortiori argument—destructive scheme (negative):

P1. *P* implies more *R* than *Q* implies *R*.
P2. *P* does not imply enough *R* to imply *s*.
C. *All the more*, *Q* does not imply enough *R* to imply *s*.

As it was mentioned before, the major premise (premise 1) could be expressed in the inferior style ("*Q* implies less *R* than *P* implies less *R*"), as well as antecedental mode can be transposed into consequental ("More *R* is required to imply *P* than to imply *Q*"). This however has no impact on the scheme. However, two questions should be investigated more carefully: Should the AFA schemes always include 4 terms?; Do these terms consist of one predicate and three arguments?

4.1.1 Does AFA Include Three or Four Terms?

Answering the first question let us consider the following example:

(f) Omniscient is more likely to belong to gods than to humans. Omniscience does not belong to gods. All the more, omniscience does not belong to humans.

If we put that argument into a list we receive[23]:

1. Omniscient is more likely to belong to gods than to humans.
2. Omniscience does not belong to gods.
3. *All the more*, omniscience does not belong to humans.

Prima facie it looks like a destructive AFA: there are two objects or class of objects (gods, humans) which differ quantitatively in regards to omniscience, and there is an all-the-more—conclusion, to which the certainty "is transferred" from the premises with even greater strength. However, this seems incompatible with the schemes we discussed earlier, since it includes only 3 terms (gods, humans, omniscience)—the scheme of the reasoning is the following (in the parentheses we include the assertions in the subjective structure):

[23] The major premise has a predicative order, which causes further problems in understanding—in subjective structure, it reads: "Gods are more likely to be omniscient than humans are; gods are not omniscient; therefore, all the more humans are not omniscient."

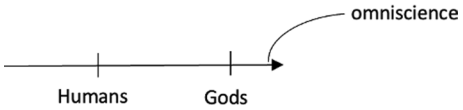


Fig. 8 Diagram validating AFA on omniscience (incomplete)

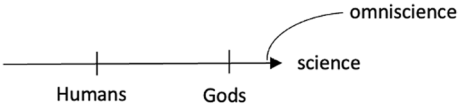


Fig. 9 Diagram validating AFA on omniscience (complete)

1. *S* is more likely to belong to *Q* than *P* (*Q* is more likely to be *S*, than *P*).
2. *S* does not belong to *Q* (*Q* is not *S*).
3. *All the more*, *S* does not belong to *P* (*P* is not *S*).

This reasoning seems valid, and it is valid indeed—however, something is missing here. Let us picture it in the diagram as we did earlier (Fig. 8):

We include humans and gods on the diagram in a way that expresses the major premise, and then we mark omniscience further on the x-axis, so it does not belong to the gods, as minor premise states. From here we see that it does not belong *even more* to the humans, which validates the reasoning. However, we might ask, what is the x-axis anyway? This is what brings us to the fourth term, which was assumed in the argument, but not expressed—this is a property of "science" (knowledge), which is the base for the comparison between humans and gods (Fig. 9):

If we make this unstated term explicit from the beginning it would look like this (we will use subjective mode):

1. Gods are more likely to be scient than humans are (*Q* is more *R* than *P* is *R*).
2. Gods are not scient enough to be omniscient (*Q* is not enough *R* to be *S*).
3. All the more, humans are not omniscient (*P* is not enough *R* to be *S*).

Analysis of any example of AFA which seems to use only three terms reveals that there are always four terms involved, but sometimes one of them (*R*)—the property which is quantitative in regards to the extreme terms—is unstated. This example brings us, however, to the second question:

4.1.2 How Many Predicates Does AFA Include?

In the aforementioned analysis and classification of AFA, we said that the argument consists of four terms: the extremes (*P* and *Q*), which are always the arguments of the predicate *R*—a quantitative property that both extreme terms share, and the fourth term, *S*. But what is *S*? A predicate or the third argument of *R*? In example (c) there were 3 arguments (triathlon, Ironman race, rowing) and 1 predicate (requiring determination), yet in examples (a), (b), and (d), there were 2 arguments and 2 predicates.[24] It is interesting, however, that the diagrams checking the formal

[24] (c)—2 arguments (students, adults); 2 predicates (being required to pay for the transport, being allowed to use a regular Oyster card); (d)—2 arguments (Brexit, restriction of the immigration policy); 2 predicates: (being supported by British society, being introduced by the government);(f)—2 arguments

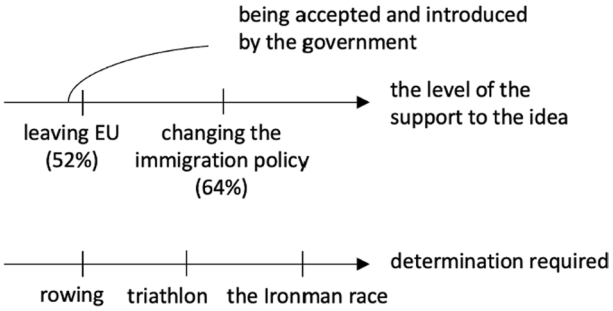


Fig. 10 Diagrams validating the arguments (**d**) and (**e**)

validity would look differently depending on how many predicates/arguments there are (Fig. 10):

There is no problem with including both schemes (with 2 predicates as well as with 1 predicate) in the concept of AFA. In both cases the core of the argument is the same: two terms (*P* and *Q*) are compared from the perspective of the quantity of the property (*R*) and the relation between *Q* (or *P*) and *S*—based on these two premises the conclusion asserts the relation between *P* (or *Q*) and S. Additionally, as we already said, predication can be understood as a very special case of implication, so the two schemes, constructive and destructive, in the antecedental structure include schemes with 2 predicates and 1 predicate as well. Therefore, they can be treated as generic AFA schemes with no more concerns.

4.2 Argument from Analogy and A Fortiori Arguments

The next thing to be considered in terms of refining the concept of AFA for argumentation theory is its relation to the well-elaborated arguments from analogy. As was already said several times, AFA is based on a comparison between two objects (or propositions) regarding the property, which differs them quantitatively. It is an analogy of a special kind then—and therefore should be incorporated into the argumentation theory as a subtype of arguments from analogy.

To prove this let us have a look at the schemes of arguments from analogy—let us consider the scheme proposed in *Argumentation Schemes* (2008, 315)—we present it with changed variables to demonstrate the similarity with the AFA scheme[25]:

Footnote 24 (continued)
(humans, gods); 2 predicates (omniscient, "scient"—in this example, the former was the higher degree of the latter).

[25] We are aware that there has been a significant number of studies made on analogical reasoning in the last two decades—for the most systematic overview see: (Bartha 2024). However, the scheme, which we follow here is still used in contemporary studies.

Similarity Premise: Generally, *P* is similar to *Q*.
Base Premise: *S* is true (false) in regard to *P*.
Conclusion: *S* is true (false) in regard to *Q*.

Comparing any of AFA's schemes (constructive or destructive) we will see that the major premise—the one that settles thecompared objects *P* and *Q* on the same x-axis (that is *R*)—resembles the similarity premise which compares *P* and *Q* (insome, relevant for the argument, context). Also, the second premises ("base premise" in the analogy) are almost identical:"P is S " and "P is R enough to be S. " The conclusions are also similar: "*Q* is S " and "*Q* is enough *R* to be *S* ").

The difference is that the argument from analogy sets a kind of resemblance between the objects *P* and *Q* (based on some sets of properties), whereas AFA sets *P* and *Q* in some hierarchical relation ($P > Q$ or $P < Q$). One may argue that the AFA includes (as it was demonstrated) four terms, and analogy includes only three—that is, there is no *R* property, which links *P*, *Q*, and *S*. However, in argument from analogy this "property" is a set of properties that makes *P* and *Q* similar—which constitutes a kind of fourth term that establishes the comparison.[26]

4.3 Critical Questions for Assessing AFA

Having said this, we may move to the last question—how to assess AFA? Since we consider AFA as a subtype of arguments from analogy, we could follow its CQs and adapt them to AFAs (Walton, Reed, and Macagno 2008, 315; Groarke and Tindale 2013, 286).[27] Thus, the first two questions refer to acceptability of the premises.

CQ1: Does *P* imply *R* more (or less) than *Q* implies *R*? / Is *P* more (or less) *R* than *Q*? (ad P1)

As it was mentioned, the predicative form could be treated as a special kind of implicational (since every inclusion/predication can be expressed as an implication). However, since AFAs are commonly expressed in the predicative form, we also provide the CQ adjusted to the predicative form.

Answering this question might be problematic though since we encounter a serious philosophical problem here. The argument assumes that there is a certain hierarchy that we could use to "measure" the level of *R* in *P* and *Q*. But how do we "measure" that? There are some areas in which we can—e.g. in legal or financial discussions:

[26] It is well-expressed in the alternative scheme for arguments from analogy: "Premise 1: X is *p, q, r,.., z*. Premise 2: Y is *p, q, r,....* Conclusion: Y is *z*." (Groarke and Tindale 2013, 281)—the properties "*p, q, r,...*" is a set of properties that makes X and Y similar, and it must be relevant for the property z that is transferred from X to Y in conclusion.

[27] For the general concept of critical questions, their functions, grammatical structure (open, closed, positively or negatively aimed etc.), distinctions from counterarguments, and their use in the discussion, see: (Van Laar, Krabbe 2013; Hoppmann 2009, 2013). We therefore propose to understand them as a kind of "procedure."

(g) Since the minimum penalty in the UK for causing death by dangerous driving is 14 years to life imprisonment, and murder is certainly the bigger crime, therefore the penalty for murder must be more than 14 years of custody.

The hierarchy of crimes and punishments was already settled by English criminal law, which makes no problems with "measuring" the harmfulness of the act and the length of the sentence. However, the problem arises with e.g. ethical discussions: how to "measure" immorality of the actions?[28] More generally, we can ask how to measure and compare the strength of the inference between "$P \rightarrow R$" and "$Q \rightarrow R$"?

To solve this puzzle we may turn to Stephen Thomas' idea of determining the strength of the inference in argument evaluation. Thomas' idea is to evaluate the inferences not only to determine if it is deductive but to scale them from less to more acceptable (irrelevant, weak, moderate, strong, deductive).[29] Inspired by this idea we could formulate the answer in the following way—for every two inferences we can ask which implication seems stronger. The fact that we could express every AFA in implicational form makes it easy from here: we simply ask ourselves if P implies R more than Q implies R. To put it simply: for any two inferences we may ask, which one of them seems more plausible—in this way we can "measure" every inference and skip the process of "quantifying" or "scaling" the property R (which in fact would be a complex philosophical problem).

CQ2: Does Q imply (or not) S because of R? / Does Q have (or have not) property S because of R? (ad P2)

The second question refers to the acceptability of the second premise. In this way, we also ensure that the property R is relevant for concluding S (in assessing the arguments from analogy the relevance of similarity to the transferred property is an additional criterion). After determining the acceptability of two premises and the relevance to the conclusion only one question remains:

CQ3: Are there any significant differences (concerning S) between P and Q?

This question is obligatory in any kind of analogous reasoning, since any pair of objects (if they are not the same) differ somehow, and this difference might be significant in the context of a property S that is transferred from P to Q. However, there

[28] It might be possible if everyone adopts the same ethical position (e.g. utilitarianism), but usually, this is not the case. This refers to broadly debated problem of validity between descriptive claims (like facts) and validity of normative claims (ethical norms). See more: (Habermas 1981).

[29] Thomas proposed the following five degrees in determining the strength of the inference: (1) P provides no support to C at all; (2) It is very probably that P and not-C; (3) P supports C, but does not prove it beyond any reasonable doubt; (4) It is improbable, but still possible, that P and not-C; (5) it is logically impossible that P and not-C (1973, 123–28). However what is interesting for us here is the idea that we can compare two different inferences and decide what is more plausible (or—using Perelman's concept—what "universal audience" would accept more [Perelman and Olbrechts-Tyteca 1969, 35]).

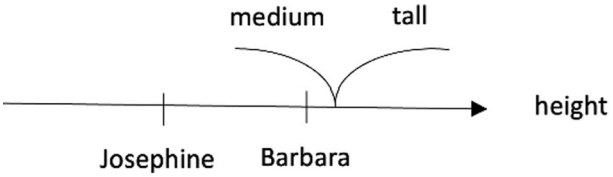
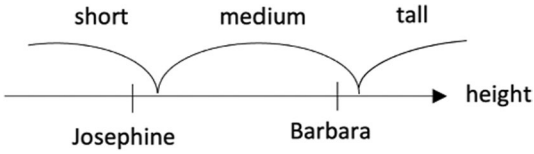


Fig. 11 Diagram validating the argument (h)

Fig. 12 Diagram invalidating the argument (h)



is one difference, that is a level of *R* in *P* and in *Q*, which needs more careful examination—let us start with the example:

(h) Barbara is taller than Josephine, yet she is still of medium height. Therefore, all the more Josephine is of medium height.

This reasoning could be evaluated with the method introduced earlier on—there are two objects here (Barbara and Josephine), and two predicates (tall, medium), which are presented at the Fig. 11:

Prima facie this argument might seem valid—however, after more critical investigation may prove otherwise. By assuming that Josephine is 5 feet tall, and Barbara is 5 feet 4 inches tall, we could easily contradict the conclusion—and this makes the reasoning invalid. Let us use the same method to demonstrate it (Fig. 12):

This shows a crucial trait of AFA—some predicates (S) may be limited "on both ends," and determining that a given object (Q) lies within its limits requires making sure that it shares the minimum level of the needed property (R), but does not exceed the maximum level of it. Therefore, we need to formulate one more critical question for the diligent evaluation of AFA:

CQ4: Is there any upper (or lower) limit of *R* that something must meet to imply *S*?

After careful examination of these four critical questions, we may determine if the argument is sound—CQ1 and CQ2 refer to the acceptability of the premises (additionally the latter include the criterion of relevance of the property *R* to *S*). CQ3 covers all significant dissimilarities between *P* and *Q* that may undermine the analogy. Finally, CQ4 ensures that the given object indeed fits (or does not fit) within the range of the predicate *S*.

5 Conclusion

In this paper, we tried to investigate the history of the concept of a fortiori argument, its components, structure, types, and methods of evaluation. The model of a fortiori arguments proposed here includes the scheme, classification, and critical questions for assessing such arguments. Additionally, we developed the diagrammatic method of validating the inference in AFAs, which may prove useful in more complex cases.

We have also managed to solve the problem of quantifying or "measuring" the level of property R in the compared objects by transforming all propositions within AFAs into implicational form. In this way, one may compare the strength of inference between the implications, which allows one to skip the problem of comparing objects or properties of different categories.

Having this model elaborated we can not only incorporate a fortiori arguments into contemporary argumentation theory and textbooks for critical thinking, but we could also include AFA into software analysing argumentation (e.g. *Online Visualisation of Argument*), and carry out the annotation and corpus analysis of AFAs on big data of the examples from the public debate.

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